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Pulsar Demography and Neutron Star Astrophysics

S. R. Kulkarni

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Pulsar demography and neutron star astrophysics

By S. R. Kulkarni

Division of Physics, Mathematics and Astronomy, Caltech 105-24, Pasadena, California 91125, U.S.A.

Pulsars, in particular, millisecond and binary pulsars offer new and unique tests of physics. The explosive birth of neutron stars and coalescence of binary neutron stars are prime targets for planned gravitational wave interferometers. All these applications require a thorough understanding of the statistics of pulsars to reliably estimate detection frequencies and thresholds. There are many aspects of the origin and evolution of neutron stars that benefit from an appreciation of pulsar phenomenology and statistics. Theoretical understanding of the magnetic field evolution of neutron stars is, at the present, poor. Statistical studies continue to play an important role in guiding us empirically in this area. The shortest spin period of a neutron star directly constrains the equation of state of dense matter. The limitations of current millisecond pulsar searches in this regard is thoroughly reviewed. The prospects and problems of searchers for pulsars with periods below a millisecond are reviewed.

1. Introduction

The collapse of stellar cores to neutron stars, the high density of neutron star interiors, the high magnetic field strengths, and the fantastic rotational stability of millisecond pulsars allow a number of opportunities to use pulsars as laboratories for studies in fundamental physics. Pulsars in orbit around other degenerate bodies open new possibilities: fundamental tests of general relativity (GR), determination of masses of planets, white dwarfs, neutron stars, and black holes. The explosive birth of neutron stars and double neutron star systems decaying by emission of gravitational waves are the prime targets for the planned gravitational wave interferometers.

The applications and opportunities discussed above are unique and in most cases the information or the signal is unobtainable except via radio pulsars. However, unlike the laboratory oriented physicist, the astronomer must be content to accept the situation as nature has designed with no possibility for rearranging the apparatus. Fortunately, these are not severe limitations. Multiwavelength observations and a good knowledge of the formation and evolution scenarios provide sufficient empirical if not full understanding of our apparatus. The statistics or the demography of pulsars show that the sky probably contains a large number of objects, the totality of which allow us to carry out a large number of tests.

The organization of this paper is as follows. The formation and evolution of binary and millisecond pulsars is first discussed (§2). The statistics of millisecond pulsars, crucial to the detection of a cosmic gravity wave background, and the importance of the limiting period of neutron stars are discussed in §3. The very important issue of magnetic field strengths of pulsars is discussed in §4. This topic is of interest from

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both the point of view of physics and astrophysics. Our theoretical understanding of the formation and the evolution of neutron star magnetic fields seems to be quite weak. Consequently, statistical studies have historically played an important role in identifying key clues. In §5, the statistics of coalescing double neutron star systems and asymmetrically exploding neutron stars; prime targets for planned gravitational interferometers are discussed. Owing to space limitations, the discussion is, unless otherwise stated, restricted to pulsars in the disc of the Galaxy.

As this is not meant to be a complete review, references will be made to recent reviews and to some key papers (Manchester & Taylor 1977; Ögelman & van den Heuvel 1989; Ventura & Pines 1991; Bhattacharya & van den Heuvel 1991).

2. Binary and millisecond pulsars: formation

Neutron stars are the stellar remnants of massive stars (mass $M > 8M_{\odot}$) and are believed to be born with relatively short periods, $P \lesssim 100$ ms, and large magnetic field strengths ($B \gtrsim 10^{12}$ G; equation (1)). The rotational energy is believed to be radiated away in the form of a pulsar wind, a mixture of relativistic particles, magnetic fields, and high energy photons. Assuming that the spin down can be adequately described by magnetic dipole radiation from an orthogonal magnetic rotating dipole, the magnetic dipole field strength is given by

$$B_{12}^2 \sim P\dot{P}_{-15}; \tag{1}$$

here $B_{12} = B/10^{12}\ G$ and $\dot{P}_{-15} = \dot{P}/10^{-15}\ \mathrm{s}^{-1}$. About 500 single pulsars, the 'garden variety' pulsars with typical periods $\lesssim 1$ s and typical characteristic ages, $\tau_c = P/2\dot{P} \sim 10^6$–$10^7$ years, have been detected as a result of systematic radio surveys of the sky (figure 1). Note that τ_c is an upper limit to the true age and is almost equal to the true age as long as the pulsar has spun down significantly and neutron star magnetic fields do not decay.

The very interesting group of millisecond and binary pulsars (figure 1) are thought to result from the evolution of binary stars. This group can be further divided: (1) high mass binary pulsars (HMBPS) which have either a neutron star or a massive white dwarf as a companion, and (2) the low mass binary pulsar (LMBP) group which have low mass ($\lesssim 0.3M_{\odot}$) companions; the single millisecond pulsars and pulsars with very low mass (planet mass) companions are considered to be a part of this group. We define millisecond pulsars to be those with $P < 10$ ms, a definition which ought to exclude all young pulsars, and also HMBPS (figure 1; see also §4*d* for rationale).

Consider a pair of massive stars ($M_1, M_2 > 8M_{\odot}$) with sufficiently small orbital separation as to allow the possibility of mass transfer at some later stage. The initially more massive star, the primary, evolves, expands and transfers matter to the secondary; eventually it explodes leaving a neutron star remnant or rarely a black hole. In a large number of cases, the system survives the sudden mass loss (Bailes 1989). The secondary evolves on a $\lesssim 10^8$ year timescale and expands. The gas from the secondary overflows its Roche lobe with attendant spinning up of the neutron star (see equation (2)) and pulsed X-ray emission. Mass transfer from the massive secondary to the less massive ($1.4\,M_{\odot}$) neutron star secondary results in orbit contraction, for the second time. For initial orbital period $P_b \gtrsim 1$ year, the secondary can swell up and become a giant before undergoing Roche lobe overflow. The gravitational potential energy released by the neutron star as it spirals towards the companion is sufficient to eject the loosely bound envelope of the giant

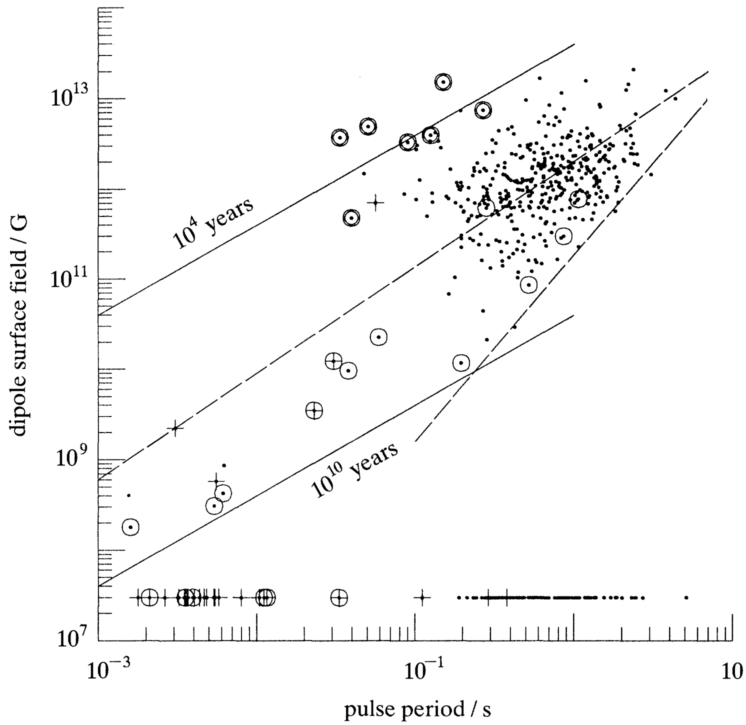

Figure 1. Dipole magnetic field strength (B) against pulse period (P) for all known pulsars. Pulsars whose period derivative has not yet been measured are shown at the bottom of the plot. ⦿, SNR; +, cluster; ⊙, binary.

secondary. This process is referred to as common envelope evolution. A binary with orbital period between a fraction of day to weeks is formed. Common envelope evolution is not well understood and, at the present time, we are unable to predict the final P_{b} given an initial P_{b}. Depending on its mass, the secondary core evolves into a degenerate remnant and the final result is: (i) a pair of unbound neutron stars (most likely; most of the dots in figure 1), or (ii) a highly circular massive white dwarf-neutron star binary, e.g. 0655+64, or (iii) a bound highly eccentric double neutron star system, e.g. 1534+12, 1913+16, 2304+46, or (iv) an eccentric-neutron-star–black-hole system (least likely). The most interesting systems are the last two and play an important role in tests of GR and probably constitute the most important sources of gravitational waves (GW) for GW interferometers (§5).

For $P_{\mathrm{b}} \lesssim 1$ year, the secondary can only swell up to modest radius ($\lesssim 1$ AU) before unstable mass transfer begins. The smaller radius, compared to the previous case, results in the secondary having significantly larger binding energy. Consequently the spiral-in of the neutron star is unable to eject the envelope. The neutron star sinks to the centre, accretes matter and eventually emerges as a slowly rotating single neutron star.

Millisecond pulsars are supposed to descend from low mass X-ray binaries (LMXBs): binaries in which a neutron star is accreting from a low mass ($M \lesssim 1\,M_{\odot}$) companion. About 50 LMXBs have been identified mainly towards the interior of the Galaxy with a dozen in the globular cluster system. These binaries are the brightest X-ray sources in the sky and have inferred accretion rates between $10^{-3}\,\dot{M}_{\mathrm{E}}$ to $\dot{M}_{\mathrm{E}}$, where $\dot{M}_{\mathrm{E}}$ is the Eddington rate, $\approx 10^{-8}\, M_{\odot}\, a^{-1}$. The orbital periods of LMXBS range from a fraction of an hour to a year.

LMXBS with $P_{\mathrm{b}} > 1$ day evolve by conservative mass transfer from the companion to the neutron star, driven by the slow expansion of the companion due to nuclear burning. Owing to its large specific angular momentum, the matter from the secondary, settles into an accretion disc around the neutron star. The inner edge of the disc is terminated at the Alfvén radius (the radius at which the accretion pressure balances the magnetic pressure). Thus the limiting period of the pulsar is the keplerian period at the inner edge of the disc:

$$P_{\mathrm{eq}} = (1.68\ \mathrm{ms})\, B_9^{\frac{6}{7}} (\dot{M}/\dot{M}_{\mathrm{Ed}})^{-\frac{3}{7}}. \qquad (2)$$

Equation (2) is usually referred to as the 'rebirth' line since old spun-down pulsars are given a fresh lease of life as radio pulsars through this process (see also figure 1). Note that (2) is really a dimensional equation and was arrived at by making several simplifying assumptions, e.g. spherical accretion, pure dipolar field, etc. We have chosen the arbitrary constant to be 1.68 ms so that the youngest millisecond pulsar lies on the rebirth line. Conservative mass transfer results in orbit expansion and the end result is a wide binary consisting of a low mass ($\lesssim 0.3\, M_{\odot}$) white dwarf and a rapidly spinning pulsar, e.g. 1953+29 and 1855+09. The pulsar lies on the rebirth line only if sufficient matter (ΔM) has been transferred to spin it up to the equilibrium period specified by its magnetic field strength (equation (2)), e.g. $\Delta M = 0.1\, M_{\odot}$ to spin up a neutron star to $P = 1.5$ ms. As the pulsar ages, it loses its rotational energy and moves horizontally rightwards.

LMXBS with $P_{\mathrm{b}} < 1$ day undergo a dramatically different evolution. Angular momentum losses – gravitational radiation and magnetic braking (i.e. loss of angular momentum via a stellar magnetized wind) – drive the two stars closer. Cessation of mass transfer may cost the companion dearly: the spun up neutron star, free from infalling matter, may turn on and function as a millisecond pulsar and start ablating the companion, e.g. 1957+20 (Fruchter *et al.* 1988). The ablation may itself drive a large angular momentum loss as is observed in 1957+20 (Ryba & Taylor 1991). A second episode of accretion may then commence and the final result could well be a single millisecond pulsar like 1937+21.

The above model with the pulsars being neutron stars which are the stellar remnants of massive stars, subsequently spin up by accretion, can be referred to as the standard model. An entirely different mechanism can be accretion induced collapse (AIC) of massive, accreting white dwarfs pushed over the Chandrashekar limit. The resulting neutron star is postulated to be born with suitably low magnetic field strengths, small periods and small velocity kicks (Michel 1987; Chanmugam & Brecher 1987). Much of the original justification for AIC has now disappeared: magnetic fields appear not to decay, LMBPS with large systemic velocities have been observed and binary pulsars intermediate to LMXBS and LMBPS have now been discovered in the disc and in the cluster system. In addition, the viability of AIC mechanism is doubtful for a variety of reasons. For all these reasons, we no longer discuss this model (see Verbunt (1992) for a summary).

3. Millisecond pulsars: statistics and limiting period

The first millisecond pulsar, 1937+21, with a period of 1.56 ms, was discovered exactly one decade ago (Backer *et al.* 1982). This discovery revitalized pulsar astronomy by showing that a very large parameter space in period was missed in earlier pulsar searches. The discovery also linked X-ray astronomy and pulsar astronomy and in particular highlighted the LMXB–LMBP connection.

There are several reasons for the great interest in millisecond pulsars:

1. As probes of a number of physical phenomena: gravity wave background, interstellar medium, planetary ephemerides and fundamental astrometry (see Backer & Kulkarni (1990) for a recent review). These applications rely on the fantastic temporal stability of millisecond pulsars. Particularly exciting is the use of an array of millisecond pulsars to detect the predicted gravity wave background. The basic idea is to treat Earth as a free mass and to use precision pulsar timing to measure local space time distortions (Detweiler 1979). Such an interferometer will be sensitive to long-wavelength gravity waves, $\gtrsim$ light years. This issue is discussed more in detail by Taylor (this symposium).

2. As probes of the physics of neutron star interiors. The maximum rotation rate of a neutron star sets very interesting constraints on the equation of state of neutron star material. A 10 km radius neutron star cannot rotate faster than 0.2 ms because the surface speed will exceed the speed of light. Instability to centrifugal forces result in a larger minimum period,

$$P_{\min} \gtrsim 0.5 \text{ ms to } 1.5 \text{ ms} \qquad (3)$$

depending on the equations of state for dense matter (Friedman *et al.* 1984; Shapiro *et al.* 1984).

Looking at figure 1, the coincidence of the periods of PSR 1937+21 and PSR 1957+20 suggests that $P_{\min} \sim 1.5$ ms. If so, hard equations of state, which predict $P_{\min} \lesssim 1.5$ ms are indicated. However, this conclusion may be quite premature, tempting it may be. None of the pulsar searches have significant sensitivity for $P \lesssim 1$ ms and are marginal even for $P = 1.5$ ms. Thus there are no real observational constraints on sub-millisecond pulsars. Discovery of even one $\lesssim 1$ ms pulsar would rule out most of the hard equations of state. Discovery of even one pulsar with $P < 0.5$ ms would rule out all proposed equations of state.

Other types of instabilities may well govern $P_{\min}$. More than 20 years ago, S. Chandrashekar showed that gravitational radiation could induce a non-axisymmetric instability in rapidly rotating stars. Various authors have speculated that this instability will raise the above $P_{\min}$ by 10–30% (see N. K. Glendenning, unpublished work; Lindblom 1992). Thus there could well be several accreting stars for which the angular momentum gain is balanced by the loss via gravitational radiation (Wagoner 1984). The expected gravitational wave amplitude is $h \lesssim 10^{-27}$, beyond the reach of most planned GW detectors. Viscosity can damp out this instability. Indeed, recently Lindblom & Mendell (1992) have argued that this instability is effectively suppressed by the viscosity due to mutual friction between the electrons and quantized neutron vortices.

The minimum period constrains the mean density of the rotating neutron star. Neutron stars with 0.4 ms $< P <$ 0.5 ms would have such high density that the nucleons are dissolved into quarks and can be referred to as hybrid stars. Glendenning has argued that ‘if pulsars with periods below 0.4 ms were found, the conclusion that the confined hadronic phase of nucleons and nuclei is only metastable would be almost inescapable.’ Strange matter has been suggested to be a plausible ground state. Thus we can see that millisecond pulsar searches can potentially throw much light upon fundamental physics.

3. As the (potential) end products of LMXBS. The hypothesized evolutionary link provides new insights into the evolution of millisecond pulsars and accretion theory. Evolutionary scenarios can be confronted by comparing the orbital parameters of the LMXBS with those of LMBPS. The precision of millisecond pulsar timing allows a number of parameters to be deduced, usually unobtainable in other ways: eccentricities, masses of companions and neutron stars, ages, distances and kinematics. Thus delicate comparisons can be made with theory. For example, the eccentricities of binary pulsars, especially in clusters, are not well understood. This has led to several theoretical studies which are discussed by Phinney (this symposium). Another example is the discrepant birthrates between LMXBS and LMBPS (Narayan *et al.* 1990), which can be reconciled by decreasing the mass transfer timescale. Reconciling this requires the X-ray phase to be significantly shorter than that predicted by simple models. This is attributed to the increase in mass transfer caused by some of the X-ray emission from the accretion disc (or spin-down luminosity of the pulsar) impinging on the companion. It is a rather complicated physical problem involving radiative transfer, cooling functions and stellar structure and this topic is now a very active area of theoretical astrophysics.

At the present, largely as a result of undirected searches, a total of seven millisecond pulsars have been discovered in the disc of the Galaxy. All but two are in binary systems. A complete list of millisecond and binary pulsars discovered to date can be found in the accompanying article by Phinney and are not reproduced for reasons of space economy. (This list is already out of data since almost half a dozen new millisecond pulsars have been discovered following the meeting!) These pulsars are already making contributions to the various applications listed above and are discussed by Taylor.

It has been estimated that the Galaxy contains about 10^5 LMBPS (Narayan *et al.* 1990; Johnston & Bailes 1991), comparable with the number of ordinary pulsars (Narayan 1987). LMBPS with their rapid spin and low magnetic field strengths can shine in the radio window for a time approaching the Hubble lifetime, 10^{10} years. The long age combined with their significant space velocities means that the millisecond pulsar layer is several kiloparsecs thick and thus millisecond pulsars should be found even at high Galactic latitudes, unlike the standard pulsar which are confined to the galactic plane. Indeed, A. Wolszczan and D. Nice have already discovered two nearby high-latitude pulsars, using the Arecibo telescope. Currently, there are several pulsar searches being carried out at Arecibo, Jodrell Bank and Parkes. Johnston & Bailes (1991) predict 80 pulsars to be detected spread essentially uniformly over the whole sky and with flux density at 400 MHz greater than 2 mJa.

Globular clusters have been proved to be very fertile hunting grounds for millisecond pulsars. Since the first discovery in 1987 (Lyne *et al.* 1987), more than 30 pulsars have been discovered (see Phinney & Kulkarni (1992) for a review). These pulsars provide important information about the structure and evolution of clusters, the tidal capture process, etc. These applications are reviewed by Phinney (this symposium). However, most of the pulsars are faint and are thus typically not useful for tests of GR or the detection of GW.

(*a*) *Searches for submillisecond pulsars: problems and prospects*

In (2) above, we argued that fundamental advances in physics can potentially be made by the discovery of pulsars below 1 ms. Here we discuss the possibility and problems of searches for submillisecond pulsars, pulsars with periods close to P_{min}.

Pulsar searches consist of obtaining n-channel power spectra spanning a bandwidth B at a spectrum sample rate of f_s (see Backer & Kulkarni 1990). The pulsar signal, owing to dispersive transmission through the interstellar medium, arrives earlier in the higher sky frequency channels compared to the lower sky frequency channels. The first step is to undo this dispersion. This is effected by suitable shifting in time of the data in each channel. If the amount of dispersion is not known, as is the case for pulsar searches, then one needs to try a variety of such shifts. Next, a search for a pulsed train need to be carried out in the dispersed time series. This is most easily implemented by a Fourier transform followed by a search for a pattern of evenly spaced peaks (the fundamental and several harmonics).

In the usual search scheme there are three different time constants, each of which must be smaller than P_m, the minimum period to which the search is designed to be sensitive. The spectrum sampling interval is

$$\Delta t = f_s^{-1}. \tag{4}$$

The smearing of the signal across one channel of frequency width, $b = B/n$, due to interstellar dispersion is

$$\Delta t_s = \alpha(b/\mathrm{MHz}), \alpha = 8.3\ \mu\mathrm{s}\, M_D (f/\mathrm{GHz})^{-3}; \tag{5}$$

where f is the central frequency of the observing band and M_D is the dispersion measure, the column density of thermal electrons to the pulsar (units: cm^{-3} pc). The rise time of the channel is

$$\Delta t_f = 1/b; \tag{6}$$

intensity fluctuations on timescales $\lesssim \Delta t_f$ are suppressed. Multipath propagation results in an effective time constant,

$$\Delta t_{ISS} \propto f^{-4} M_D^{+3} \tag{7}$$

(Cordes *et al.* 1985); the two exponents are approximate. Thus the effective temporal resolution of the dedispersed time series is

$$\Delta t_e = \surd(\Delta t^2 + \Delta t_s^2 + \Delta t_f^2 + \Delta t_{ISS}^2). \tag{8}$$

A pulsar survey is sensitive to pulsars with $P \gtrsim P_m \sim \beta \Delta t_e$. Normally, $\beta = 2$ is considered sufficient to detect one harmonic. However, most millisecond pulsars appear to have an interpulse separated by about half a period. This results in the cancellation of most odd harmonics. In addition, due to radio frequency interference, it is important to detect at least one and preferably several harmonics. Thus $\beta \gtrsim 8$.

Let us now consider a specific example. Millisecond pulsars have steep radio spectra making it advantageous to search at metre wavelengths. So let us set $f = 400$ MHz. Restricting the search to about 1–2 kpc radius ($M_D \lesssim 30\ \mathrm{cm}^{-3}$ pc; t_{ISS} can be ignored for nearby pulsars) the minimum value of Δt_e is

$$\Delta t_e > \surd(2\alpha) > 88\ \mu\mathrm{s}; \tag{9}$$

this assumes $\Delta t \ll \Delta t_e$. Note that $\Delta t_e \gtrsim \frac{1}{8} P_{min}$. Inequality can only be achieved by operating at a higher radio frequency. Changing to $f = 610$ MHz results in Δt_e to 46 μs, which is quite acceptable.

The condition $\Delta t \ll \Delta t_e$ results in high data rates. At such high rates, coherent dedispersion techniques can be profitably used. This requires the recording of not the intensity but the electric voltage of the signal. The real advantage of the technique is that the constraint specified by (5) is completely overcome. Δt_{e} is effectively limited only by (4) and (6). Coherent dedispersion has been applied in the studies of known pulsars (Hankins 1971). For searching, it is computationally very intensive. Recording the electric field requires high recording bandwidths, e.g. rates upwards of 2.5 Mbyte s^{-1} are needed for a modest bandwidth of 10 MHz. This is feasible but expensive. Fortunately, suitable supercomputers exist and it is now possible to undertake, at least towards intriguing targets, the search for the fastest pulsars. The unique radio signature of pulsars – scintillation, steep radio spectrum and high linear polarization – allow us to identify such targets from specially designed continuum surveys of the sky. With the dramatic rise in computing speeds, it is quite conceivable that within a few years coherent pulse searches will be routinely carried out!

The ultimate limit to the search for the most rapidly rotating pulsars will be set by t_{ISS}. Demanding that $\Delta t_{\mathrm{ISS}} < 100$ μs leads to the restriction, at 400 MHz, $DM \lesssim 100\ \mathrm{cm}^{-3}$ pc. This constraint allows us to explore the local 3 kpc radius volume quite effectively and hence is not a significant restriction. Distant parts of the Galaxy can be searched by shifting to a higher frequency like 610 MHz.

4. Magnetic field strengths of pulsars

Magnetic field strength is probably the key parameter in the life history of a neutron star. The spin down luminosity is $\propto B^2$ and the equilibrium spin-up period is $\propto B^{\frac{6}{7}}$ (equation (2)). The observational detectability of a pulsar is directly related to the field strength; the long lifetime of low magnetic field strength millisecond pulsars ensures a large population. The origin and maintenance of magnetic fields in neutron stars with their normal crusts and superconducting cores is not understood and is consequently a challenging problem to physicists. For these reasons, this topic has attracted a number of observational and theoretical studies.

It is fair to say that much of our theoretical edifice of neutron star magnetic fields has been strongly influenced by statistical analyses of a variety of pulsar observations. Consequently, I first summarize the observations and show that a systematic pattern of magnetic field behaviour is now discernable. These provide clues for those wishing to work in this field. We then consider the possibility of neutron stars with very low magnetic field strengths. The reader is referred to Bailes (1989) for a particularly seminal review. An observational summary now follows.

(*a*) Almost all young pulsars have inferred (equation (1)) dipole field strengths $B \gtrsim 10^{12}$ G. Similar values of B have been inferred by observations of cyclotron lines in accreting neutron stars with massive companions, i.e. young systems.

(*b*) There is some controversy whether the braking torque is influenced by the angle between the magnetic and spin axes. One group (Lyne & Manchester 1988) finds that this angle evolves secularly on a timescale of 10^7 years whereas another (Bhattacharya 1989) does not.

(*c*) The slow, i.e. the ordinary and older, pulsars have typically field strengths a factor of few less than the young pulsars (see figure 1).

(*d*) A clear bimodal distribution in field strengths of the LMBPS and the HMBPS is seen (figure 1). HMBPS have field strengths above 10^{10} G and LMBPS have field strengths between 10^8 G and 10^9 G. We will refer to this as the magnetic field gap. According to equation (2), a magnetic field gap results in a period gap. Thus there should be no young LMBPS with $P > 10$ ms.

(*e*) The field strengths of disc millisecond pulsars seem to have very little dispersion: $B \sim 3 \pm 1 \times 10^8$ G. (PSR 1257+12 nominally has $B = 8.8 \times 10^8$ G; however, its timing solution is not fully secure (Wolszczan & Frail 1992)).

(*f*) The magnetic fields of millisecond and binary pulsars do not evolve significantly over a Hubble lifetime (Kulkarni 1986; Callanan *et al.* 1989; Kulkarni *et al.* 1991). This inference is arrived at by noting that the white dwarf companions are cold and hence old and that the pulsar must be even older since it was born before the white dwarf.

(*g*) Although the period derivatives for most cluster pulsars have not yet been determined, the two reliable determinations that are now available place the pulsars in the middle of the field gap. The period distribution of the cluster pulsars is also different (see figure 1, bottom) from that of the disc LMBPs suggesting (via (2)) a different distribution of magnetic field strengths. It is also curious that while most of the disc millisecond pulsars have pulse profiles with several components the cluster pulsars appear to have simple profiles.

(*h*) The magnetic fields of millisecond pulsars appears to be largely dipolar. Assuming that the spin down power can be approximated by vacuum electromagnetic radiation one obtains upper limits: $\lesssim 5 \times 10^{10}$ G, quadrupole; and 3×10^{12} G, octupole (Krolik 1991). The rebirth line constrains multipole fields even more stringently albeit (accretion) model dependent (Arons 1992). In this vein we note that the cyclotron lines discussed in (*a*) measure field strengths close to the surface of the neutron star. The similarity of B deduced from (1) and the cyclotron derived values suggests that higher-order multipole contribution is not significant in young neutron stars as well.

A summary of our current theoretical and phenomenological ideas now follows. The observed small dispersion in B of young pulsars seems to be incompatible with flux freezing of the magnetic field of supernova cores and indicative of an object-independent fundamental mechanism. This has been ascribed to a generation of a dipole field anchored in the crust by a dynamo process with saturation set by crustal properties (Blandford *et al.* 1983; Thompson & Duncan 1991). The systematically smaller field strengths of the slower and hence older pulsars has been traditionally explained by postulating magnetic field decay on timescales, $\tau_{\mathrm{d}} \sim 10^7$ years. If fields do decay, then the characteristic age, τ_{c}, should be larger than the true age. The simplest test of this is to compare τ_{c} with kinematic age, $\tau_z = |z|/v_z$ where z and v_z are the vertical height and speed of the pulsar. Here it is assumed that most disc ordinary (i.e. slow period) pulsars are born in the Galactic plane and are sufficiently short lived that they do not have time to execute several oscillations. Bailes (1989) argues that the present kinematic data do not support magnetic field decay in contrast to the older studies (see Lyne 1987). Recent proper motion may be supporting Bailes's claim. A number of ordinary pulsars have now been found to be returning to the Galactic plane (Harrison *et al.* 1992).

Observation (*f*) and the existence of pulsars in globular clusters, objects believed to have been formed in the earliest epochs of the formation of our Galaxy, reinforce the view that magnetic fields do not decay in pulsars, millisecond, binary or otherwise. Indeed, from the point of view of theory, field decay has been hard to explain. While some field decay may occur in the outer crust, it has been noted that the field also diffuses inwards where the conductivity is so high that fields do not decay substantially over Hubble lifetime (Sang & Chanmugam 1987). The prevailing view now is that field decay does not take place on short timescales, $\tau_{\mathrm{d}} \gtrsim 10^8$ years.

However, given the low field strengths of millisecond pulsars, field reduction must be taking place. Clearly the reduction must have something to do with the binary aspect of the millisecond and binary pulsars. Several hypotheses have been suggested: the heating of the surface during the accretion phase, the screening of the crustal field by the accreted material, and inward advection of the crust (Romani 1990). Since most massive stars begin their lives in binary systems it is conceivable that the most of the 500 disc pulsars have gone through a modest amount of mass transfer. This explains the systematically lower field strengths of the ordinary pulsars compared with the young pulsars. Implicit in this assumption is that most young pulsars we see are not recycled. This is probably true and it is a curious and inexplicable fact in itself.

Another hypothesis notes that the magnetic field flux tubes may be pinned to the vortex tubes in the interior of the neutron star. Thus as the neutron star spins down, the field migrates radially outwards to the crust where rapid decay is postulated (Srinivasan *et al.* 1990). The last assumption seems questionable in view of the discussion above. Ruderman (1991), in a series of papers, has developed a related idea in much greater detail. In a spinning down neutron stars, the crustal plates and the magnetic field embedded in them move towards the equator. This results in a slight initial increase in B followed by a gradual decrease; $B \sim 10^{11}$ in old spun-down pulsars. Thus Ruderman's models avoid the problem of field decay in a rather clever way but at the cost of increasing the higher-order fields (see below). Subsequent spinning up as (equation (2)) moves the plates towards the poles. Substantial field reduction is possible only if the initial field configuration has both magnetic poles in the same hemisphere. Crustal motion brings the two magnetic poles to a rotational pole which results in a significant decrease in the dipole field strength. Thus the pulsar will behave like a weakly magnetized aligned rotator. However, do note that while the dipole field strength has been reduced the higher order components are not necessarily reduced. This may contradict observation (h) above. If on the other hand, as favoured by supporters of the AIC model (Michel 1987; Chanmugam & Brecher 1987), millisecond pulsars are formed by spinning down of a rapidly rotating ($P \lesssim 1$ ms) and magnetized ($B \sim 10^{10}$–10^{11} G) pulsar then the magnetic poles will quickly drift to the equatorial plane and the pulsar will effectively appear as an orthogonal rotator.

From the discussion above it must be clear that the geometry of the pulsar magnetic field with respect to the rotation axis provides a significant clue to the evolution of magnetic fields in pulsars. Some geometrical information can be obtained from pulse profiles especially in linear polarization. Unfortunately, the polarization data on millisecond pulsar profiles cannot be unambiguously interpreted (Thorsett & Stinebring 1990). All one can say is that most disc millisecond pulsars are more complicated than the average pulsars with emission throughout a large fraction of the period, traditionally taken to be a signature of an aligned rotator. However, in two of these cases, 1855+09 and 1957+20, the detection of Shapiro delay and the eclipse geometry, respectively, allows us to conclude that we are observing an orthogonal rotator. (In arriving at this conclusion, the reasonable assumption that the rotation axis is aligned with the orbital axis has been made.) Perhaps special relativistic effects (aberrations) significantly modify the underlying beam at such rapid rotation rates. Fortunately there is hope for much progress in this area given that a large number of millisecond have been (and will continue to be) discovered. We eagerly await polarimetry of the new millisecond pulsars.

(*a*) *Empirical summary of magnetic field evolution*

From the discussion above, it follows that pulsars are born with large dipole field strengths, $\approx 10^{12}$ G. These fields do not decay substantially. Mass transfer apparently leads to field reduction but with a gap centred around $10^{9.5}$ G. Most millisecond pulsars have very similar field strengths, $B \approx 10^{8.5}$ G. We conclude with a discussion of four interesting questions.

Accretion induced field reduction. Empirically, it appears that accretion leads to field reduction. What is the physical mechanism? Screening, plate motion, advection?

What is the origin of the magnetic gap? Why are there no disc pulsars with field strengths between 10^9 G and 10^{10} G? We suspect that this gap is the result of a tail in the HMBP B field distribution and the existence of a fixed magnetic field strength for LMBPs. If accretion does cause field reduction, then the distribution of the magnetic field strengths of HMBPs reflects their mass transfer histories. HMBPs evolve from massive binaries in which mass transfer is short lived. Thus it is quite possible that there are very few high-mass systems in which mass transfer is sufficiently long lived to produce very low field strength pulsars.

What is so special about 3×10^8 G*?* From figure 1 we see that a number of millisecond pulsars have magnetic field strengths close to 3×10^8 G. This gives the impression that this field strength must be of some significance. However, the observed cut-off in the magnetic field strengths could be a selection effect. Neutron stars with very low field strength pulsars would presumably be spinning at the limiting period, P_{min}, and the low field strength guarantees that they will not undergo significant period evolution over Hubble lifetime. The short period of such objects essentially render them invisible to present millisecond pulsar surveys, especially if $P_{\mathrm{min}} \lesssim 0.7$ ms. In addition, given their low bolometric luminosity, such pulsars may be faint at radio wavelengths. The relationship between B and P and radio luminosity is ill known. Stollman (1987) has argued that radio luminosity of pulsars depends upon $Q = B/P^2$ and saturates when Q exceeds 10^{13} G s^{-2}. If so, a pulsar with $B = 3 \times 10^7$ and $P = 1$ ms should be as bright as the other observed millisecond pulsars. However, it is also possible that at such low magnitude field strengths, pair cascades cannot be supported and the neutron star may not shine as a radio pulsar (Phinney 1991).

Thus our conclusions are necessarily ambiguous: either the observed millisecond pulsars are the tip of a large population of low magnetic field neutron stars or that for some unknown reason if sufficient mass is transferred then the fields are reduced to 3×10^8 G. If the latter case, then $B = 3 \times 10^8$ G must emerge from any valid theory of neutron star magnetic fields. If the former case, it could well be that the sky is full of pulsars with $P = P_{\mathrm{min}}$ and low field strengths! Clearly, pulsar searches sensitive to $P \lesssim P_{\mathrm{min}}$ are needed to explore this possibility. In addition, theoretical work is urgently needed to see whether such pulsars pulse at radio wavelengths.

Why are cluster pulsars different from disc millisecond pulsars? The period distribution of cluster pulsars is quite different from that of the disc pulsars (figure 1) and hence by inference the magnetic field strength distribution is also different. We attribute this to the different accretion history of cluster pulsars. Cluster pulsars are supposed to evolve from tidal captures whereas the LMBPs evolve from long lived binaries with a long history of mass transfer. In this sense, cluster pulsars with their presumed short and perhaps intense mass transfer phase are similar to the HMBPs!

Again, the distribution of B may only reflect their mass transfer histories to be intermediate to HMBPS and LMBPS – a plausibility given our knowledge of the formation of cluster pulsars (Phinney & Kulkarni 1992). This hypothesis does not explain why their profiles should be systematically different. However, we would like to caution the reader that most of the cluster pulsar profiles are of low quality and thus one should reserve judgement pending high quality profile observations.

5. Gravitational wave sources: statistics of coalescence and asymmetric explosions

Coalescing neutron stars are the one secure and well-understood source of gravitational waves. As such they have provided the main justification for the proposed gravitational wave interferometers, e.g. the Caltech–MIT Laser Interferometer Gravitational-Wave Observatory (LIGO), among others. As remarked in §2, massive binaries in rare ($\approx 1\%$) cases leave a pair of neutron stars (NS–NS), e.g. 1913+16, 1524+24 and 2303+46 and 2127+11C; the last is in a globular cluster. In most cases, the binary is expected to be quite eccentric because of the sudden mass loss resulting from the formation of the second neutron star. If the final orbital period is less than a day then the system will merge within Hubble lifetime. Three of these four will merge in less than 10% of Hubble lifetime.

The statistics of such systems are clearly of the greatest importance for gravitational wave astrophysics. Based on the known population and statistical analyses of the major pulsar searches, the merger rate has been estimated (Narayan *et al.* 1991; Phinney 1991) and the conclusions are optimistic.

1. The birthrate of NS–NS binaries is 10^{-3} that of the single pulsar birthrate. Despite this, the steady populations are almost comparable, $\lesssim 10^5$ (over the whole Galaxy). This comes about because of vastly differing lifetimes. Single pulsars live for $\lesssim 10^7$ years whereas in the NS–NS systems, the first born pulsar undergoes accretion with attendant decrease in B and P and hence a very long radio lifetime, $\lesssim 10^{9.5}$ years. The lifetime of most NS–NS systems is limited by the gravitational merge time, $\sim 10^{9.5}$ years.

The long lifetime and their large velocities means that the scale height of these systems is large, $\gtrsim 3$ kpc. The pulsars are expected to be faint but the predicted large number bodes well for on-going all sky pulsar surveys.

The formation rate of the NS–NS binaries has been estimated to be $\lesssim 1\%$ of the birthrate of the massive X-ray binaries, the progenitors of all HMBPS. Thus for every NS–PSR binary there should be about 10^2 unbound pairs of a young pulsar and a recycled pulsar with similar B and P as the pulsars in NS–PSR systems. It is quite disturbing that there are few, if any, such recycled pulsars in the known population.

2. A conservative merge rate of three per year within the local 200 Mpc-radius volume; an ultra-conservative rate of three events within the local 1 Gpc radius volume. These events are expected to be detected by LIGO with advanced detectors (Abramovici *et al.* 1992).

More intriguing are the black-hole–neutron-star (BH–NS) and black-hole–black-hole (BH–BH) binaries. Suggestions for the minimum main sequence mass for black hole formation have ranged from $20\,M_\odot$ to $80\,M_\odot$. Schild & Maeder (1985) have, on the basis of membership of pulsars in young clusters and a knowledge of the initial mass function (IMF), argued that all stars with $M > 50\,M_\odot$ form black holes. The high mass of the black hole primary ($\gtrsim 10\,M_\odot$) ensures that the binary remains bound

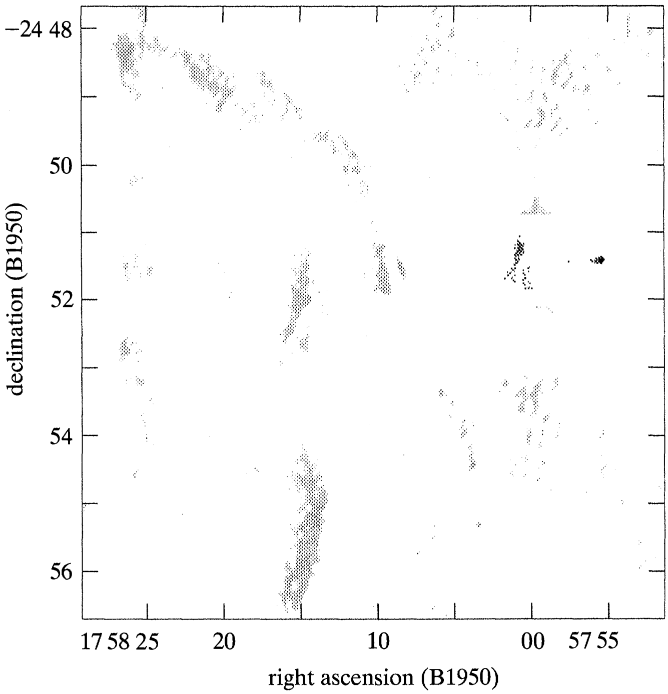

Figure 2. VLA radio image of the region around PSR 1757 − 24 (courtesy D. Frail). The morphology is strongly suggestive of a speeding bullet with the pulsar at the tip of the emission (extreme right). The wispy emission generally seen on the left side is western portion of the supernova remnant G5.4–1.2. The pulsar is believed to have been born in this remnant about 10^4 years ago.

even after the explosive birth of the secondary neutron star. (We note that the systematic radial velocities of the two better known black hole candidates are not peculiar, suggesting that black hole formation is not asymmetric, unlike neutron star formation.) For this reason, the birthrate of BH–NS systems is comparable to NS–NS systems.

The expected distribution of the orbital period of BH–NS binaries is not known. If BH–NS systems form with sufficiently short orbital periods like their NS–NS counterparts then they are equally important for LIGO-like projects. Regardless of this, BH–NS binaries will be important for tests of GR. Such binaries will be necessarily eccentric. Periastron shift and in suitable cases, Shapiro delay can be detected even from wide orbits and precise measurement of the mass of the black hole will then be possible. Measurement of higher-order GR effects will permit the verification of the existence of a black hole to a degree not possible with any other technique.

What are the prospects of finding black-hole–pulsar binaries? The pulsar will shine for a short time, $\lesssim 10^7$ years and so do the progenitors. Thus we expect to find such systems near sites of star formation. About 1 in 300 pulsars are expected to be found in such systems (Narayan *et al.* 1991). This estimate may be a bit optimistic since we know no such system in the current population of 500 disc pulsars. Nonetheless, given the importance of such systems, we should continue with deeper searches of the Galactic plane.

Other and more frequent stellar sources of gravity waves are supernovae collapse. Symmetric collapse are too weak to be detected beyond 100 kpc by LIGO-like interferometers. However, there is mounting evidence that most supernovae collapses are asymmetric. This is most easily seen in the observed space motions of pulsars which appear to range up to 10^3 km s^{-1} (Harrison *et al.* 1992) or more. The most extreme case of high velocity is perhaps PSR 1757 − 24 associated with the supernova remnant G5.4-1.2 which has been argued to have a velocity of nearly 0.5% speed of the light (Manchester *et al.* 1991; Frail & Kulkarni 1991). Three of the remnants resemble G5.4-1.2 suggesting that about 1% of pulsars have such large velocities. This fraction is also consistent with the statistics of the proper motion sample of Harrison *et al.* (1992).

Clearly the collapse has to be asymmetric to generate such large velocities. Intrinsic asymmetry in the neutrino emission seems unlikely. Collapse of spinning cores may become non-axisymmetric as the collapse proceeds. Centrifugal forces may halt the collapse of a rapidly spinning core. Further collapse requires loss of angular momentum. It has been speculated that a yet to be specified instability may drive the flattened, spinning core into a non-axisymmetric shape (see Abramovici *et al.* 1992). If gravitational waves and not hydrodynamics waves carry off the excess angular momentum then such supernova will be copious and most frequent sources of GW. If 1% of the type II supernova explosions are grossly asymmetric then there should be three events within the local 60 Mpc volume which will be easily detected by the first generation LIGO-like systems (Abramovici *et al.* 1992). It would be a great coup to both GW and pulsar astrophysics if the relationship between the two manifestations of non-axisymmetric collapses: pulsar velocity and GW amplitude can be directly related via a model. Much work needs to be done but it is already clear that pulsar astrophysics has many new surprises in store for both the physicist and the astronomer.

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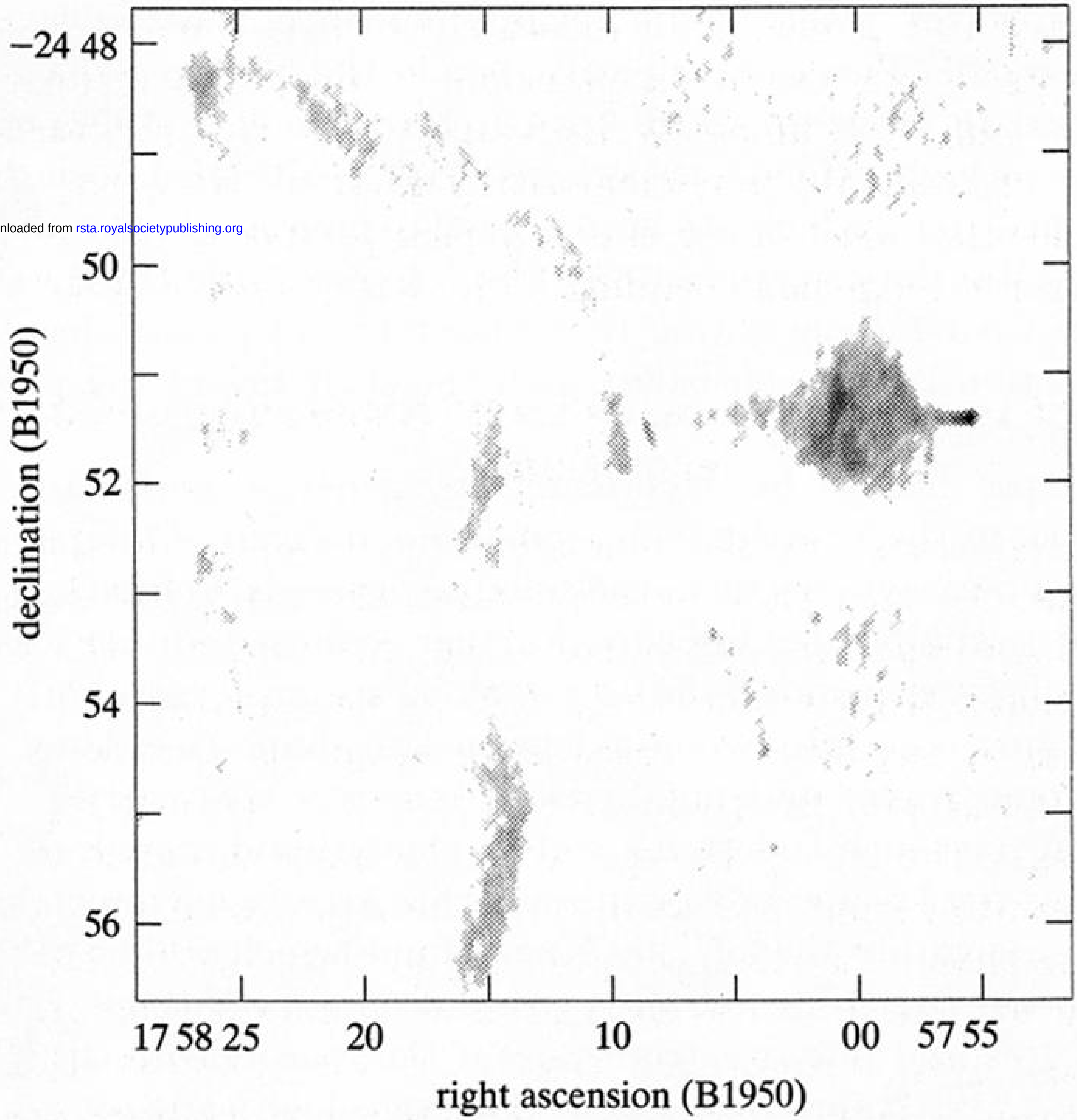

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